A Multi-Echelon, Multi-Product-Type, Site Selection and Inventory Allocation Supply Chain Model for Lean Facilities

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ABSTRACT

Effective supply chain management in today's business world is considered a competitive advantage and supply chain managers today face a myriad of inter-related decisions ranging from inventory allocation to site selection to shipment scheduling. Unfortunately, most supply chain models consider such decisions independently, and therefore miss the effects of numerous tradeoffs. Managers thus face the lack of a tool by which they may evaluate the merits of novel integrative supply chain management paradigms such as lean logistics. Here, a mixed integer non-linear programming model was formulated for a supply chain with four echelons, each with multiple sites. The first echelon consisted of suppliers, then factories for the second echelon, down to depots and cross-docks in the third echelon, and finally, customers for the fourth echelon. The model made use of one-for-one base-stock replenishment policies for each facility and it also considered different modes of replenishment from suppliers to factories, namely, the traditional direct replenishment and the lean logistics mechanism of milk runs. The model also incorporated the choice between pull mechanisms Kanban and Constant Work-in-process or ConWIP. End-product demand generated by the customer echelon was modeled as stochastic. The objective was to minimize total system costs including capital and operating expenses, holding costs, transportation costs and backorder costs. The decision variables involved selection of sites for factories, depots and cross-docks, target inventory levels, replenishment frequencies and choice of pull system.

KEYWORDS: *Supply chain management, lean logistics, inventory allocation, site selection*

I. INTRODUCTION

It has been said that competition in this age is no longer among individual companies but rather among supply chains of companies seeking to find better ways of adding value to their products en route to the final customer. A supply chain has been described as consisting of suppliers, manufacturers, distributors, and customers linked in terms of forward material flow, with a complement information flow going backward (Huang, Uppal and Shi, 2002). The science involved in considering supply chains must therefore involve all activities included in the transformation of the primary raw materials to the final product made available to the end customer. Supply chain management has been defined as "the management of materials and information both in and between facilities such as vendors, manufacturing and assembly plants, and distribution centers." (Thomas and Griffin, 1996, as cited by Huang, Uppal and Shi, 2002) Supply chain management is in itself a field that is inherently multi-disciplinary (Huang, Uppal, and Shi, 2002). The supply chain manager faces a myriad of decisions, including inventory allocation, and shipment scheduling. Hopp and Spearman (2000) point out that even the configuration of a supply network with regards to the location of factories and warehouses is already a decision on its own.

It can be seen that current research has focused mostly on approaching the issues of supply chain management independently. The problem with this approach is that it sidesteps the degree to which these decisions are all linked. Therefore, by treating them independently, current models miss a significant opportunity for cost savings.

This limitation of supply chain management models cripples the application not only of supply chain concepts in general, but also their variants, management principles that require consideration of the different aspects of supply chain management as a whole. One of these is the principle of lean logistics.

In an attempt to extend the logic of the Toyota Production System into the realm of logistics systems, Jones, Hines and Rich (1997) introduced what is today known as lean logistics or lean supply chain management. They presented a new way of viewing and managing the supply chain, in the light of the paradigm shift started at Toyota some 50 years ago. Thus, lean logistics may be described as the application of lean principles in the context of supply chain management. Lamming (1996) states that an examination of supply chain management is necessary to better understand lean production.

Thus, just as mechanisms have been developed for implementing lean within a production facility, some mechanisms have also been recommended for applying lean principles in a supply chain context. One of these is the milk run, where a routing of a supply or delivery vehicle is determined in order to make multiple pickups or drop-offs at various locations (Womack and Jones, 1996). Another mechanism would be the use of cross-docks, in contrast to setting up huge warehouses for inventory storage close to the customer. In the context of facilities planning and management, cross-docking means shipping directly from the receiving dock (Tompkins et. al., 1996). In the context of lean supply chains, the appeal of cross-dock facilities is that products are not stored but rather moved immediately from an incoming vehicle to an outbound shipping lane (Womack and Jones, 2002).

The rise of lean thinking and lean supply chains has left managers with an awful lot of factors to weigh in making their decisions. The matter becomes even more complicated when considering more complex supply networks, where there may be multiple suppliers for each part-type of a product, multiple factories, multiple depots and cross-docks, serving a dispersion of numerous customers. Although Rother and Shook (1997) have introduced a step-by-step guide to applying lean principles within production facilities, and Womack and Jones (2002) have done the same for developing lean supply chains, the fact remains that lean principles are more than a set of techniques that can be applied procedurally to any company. True, these workbooks supply managers with simple guides to improving their processes. It is however equally important to examine each aspect of lean and determine which can truly lead to better performance for the unique supply chain environment and which ones should rather be done away with. This sentiment is best resounded by Hopp and Spearman (2000) in mentioning the complexity of manufacturing in a highly competitive arena. There cannot be any "simple, uniform solution that will work well across a spectrum of manufacturing environments."

Section II discusses some existing supply chain models. Section III describes in detail the system to be modeled in terms of parameters and decisions. The general formulation is discussed in Section IV. Model validation with a specific numerical example follows in Section V. Finally, some recommendations for further study are given in Section VI.

II. LITERATURE REVIEW

There are numerous authors who have developed mathematical models in order to tackle supply chain issues. For example, multi-echelon inventory management has been investigated by Ettl et. al.(2000), Svoronos and Zipkin (1991), Aviv (2003), Sherbrooke (1967), De Bodt and Graves (1985), Gallego and Ozer (2003), Boyaci and Gallego (2001), Gallego and Zipkin (1999) and Axsater (1990). Napolitano (1997), Berman, Drezner and Wesolowsky (2001), and Viswanadham and Raghavan (2000) delve in to the issue of site selection, while Cetinkaya and Lee (2000), and Viswanathan and Mathur (1997) seek to integrate shipment scheduling and stock replenishment decisions.

This section will focus on two studies that come closest to this study in its objective of integrating supply chain decisions.

Dogan and Goetschalckx (1999) develop a model for making strategic decisions in supply chain networks. The supply chain that is modeled here includes suppliers, factories, as well as distribution centers. These make up a total of four echelons. The system involves multiple-products as well as multiple-periods, meaning a decision is made for each period as to the selection of locations to use for each type of facility.

The formulation of the objective function and the constraints are done using mixed integer programming. Costs are minimized subject to service level constraints that ensure that deterministic seasonal demand is met by the network configuration. The model is to determine the optimal configuration based on site selection, inventory allocation and transport flows.

The model of Dogan and Goetschalckx (1999) is perhaps closest to the model developed in this study. For one, it integrates decisions of site selection, inventory allocation, as well as determining which facilities are to be supplied by others. Third, it considers facilities both with production and distribution functions, thus encapsulating a more broad scope of the supply chain. Second, the range of costs considered is the most robust among all other models surveyed in the literature.

The difference however would lie in two considerations: demand and inventory policy. For the former, Dogan and Goetschalckx (1999) deal with deterministic demand, although they also consider how this demand could vary according to seasonality. In this study, however, demand will be approached as stochastic, making it more applicable to situations where actual demand exhibits more variability than simply that brought about by seasonality. Second, for Dogan and Goetschalckx (1999), since demand is deterministic, then inventory level is simply a function of the inflows and outflows to each facility. On the other hand, consideration of stochastic demand would entail determining the optimal ordering policy, when to order and how much.

The system that is modeled by Cohen and Lee (1988) is a combination of production and distribution facilities. The production facilities procure materials from suppliers and make up the upstream echelons while downstream echelons are composed of a distribution network. The supply chain follows an MRP orientation. They assume fixed facility locations and arborescent delivery arrangements between facilities.

The overall model is a composite of four different sub-models: material control, production control, finished goods stockpile, and distribution network control. Such is used to determine control policies that affect inventory control, plant product mix, and production scheduling, and is intended for use as a strategic planning tool. A decomposition method is employed such that although each model is optimized individually, control parameters set at one sub-model, or the service targets at that sub-model, affect the performance of another sub-model.

Their study is similar to this in the sense that it considers supply chains consist both of production and distribution facilities. In fact, it represents one of the first attempts at formulating a model for integrated production and distribution networks. As such, it deals with the different tradeoffs involved between inventory-control, production scheduling and product-mix decisions at the plants.

It is however different from this study because Cohen and Lee (1988) assume that production and distribution sites are fixed, whereas here, the site selection for factories, depots and cross-docks are taken as decision variables. The second major difference is that Cohen and Lee (1988) follow an MRP-framework, where each facility receives ordering and production schedules from a central control system, whereas this study considers lean networks that produce units only when triggered by customer demand for a finished product.

III. SYSTEM DESCRIPTION

III.1 System Parameters

The following notations are used to designate the general parameters in the model.

$SCAP_{sr}$	Capacity of supplier s to supply part r in units per time
$KCAP_{kp}$	Capacity of cross-dock k to distribute product p
RPT_{pf}	Raw process time of product p in factory f
$FREQ^{max}_{rsf}$	Maximum frequency by which supplier s can replenish part r via direct shipment to factory f
$TRAV_{ss'}$	Travel cost from supplier s to s' in a milk run
$TRAV_{fd}$	Travel time from factory f to depot d
$TRAV_{dc}$	Travel time from depot to customer
$TRAV_{sf}$	Travel time from supplier to factory
$BOTL_{pf}$	Bottleneck rate of a line for product p at factory f
DUE_{cp}	Time between order and expected receipt of order at customer c for product p
$SERV_{cp}$	Desired service level for product p at customer c
$CONV^{line}_{p}$	Conversion factor for a line of product p
$CONV^{store}_{p}$	Conversion factor for storage space of product p
$CONV^{horizon}$	Conversion factor for the number of time units for the planning horizon
$CONV^{day}$	Conversion factor for the number of time units in a day
$LINE^{max}_{f}$	Maximum number of standard lines to be installed into factory f
$STORE^{max}_{d}$	Maximum number of standard units to be stored in depot d
Q_{cp}	Order quantity of customer c

	for product p
REQ_{rp}	Number of units of part r that are required to produce one unit of product p
WIP_{pf}	The fixed number of units of work-in-process in a Kanban line for product p at factory f, including work-in-process at each supermarket and work-in-process at each station.

There are four-echelons in the network at hand. Shipments move downstream each echelon n containing facilities j such that $j \in ech(n)$, from suppliers s ($s \in ech(n=1)$) to factories f ($f \in ech(n=2)$) to depots or cross-docks d ($d \in ech(n=3)$), until finally to the end customer c ($c \in ech(n=4)$). For the third echelon, if d is a cross-dock, then $d \in cdock$. This allows the formulation of special constraints for cross-docks, to be discussed later, while maintaining some constraints that work for both depots and cross-docks in general.

Shipments move from an echelon n to n' such that $n<n'$. Transshipment, or shipment between facilities of the same echelon, that is $n'=n$, is not to be considered, neither is movement upstream for rework, that is $n>n'$.

Each supplier may produce one or more part-types, while there may be more than one supplier of each part-type. The set $sup(r)$ contains all suppliers s capable of producing part r. A supplier s may produce a part r only up to its capacity of $SCAP_{sr}$ given in units per time. Also, a supplier may replenish part r via direct shipment to a factory only at some maximum frequency $FREQ^{max}_{rsf}$.

A part type, meanwhile, may be used as a component of one or more products, such that $bom(p)$ is the set of all part-types r used to produce product p. Conversely, $der(r)$ is the set of all products p that can be derived from part-type r. The number of units of part r needed to produce one unit of product p is REQ_{rp}.

Factories are the only facilities where assembly and processing may take place. Products are produced on lines and each factory is capacitated in the sense that it may have a maximum of $LINE^{max}_f$ equivalent lines installed. One line of product p requires the amount of space of $CONV^{sline}_p$ equivalent lines. A line may be classified as either Kanban or ConWIP[1]. Each line assigned to a product in turn can be characterized by the raw process time RPT_{pf}, the bottleneck rate $BOTL_{pf}$, and, in the case of Kanban lines, the fixed number of work-in-process units WIP_{pf}.

Depots are storage facilities replenished by factories and from which inventory is taken in order to satisfy customer demand. Replenishment of stock at the depots follows the one-for-one replenishment policy. There is only enough space at a depot to hold $STORE^{max}_d$ equivalent units. One unit of product p requires the amount of space of $CONV^{store}_p$ equivalent units.

The cross-dock is the other facility that can directly serve the customers. Cross-docks however may not store inventory. They can only satisfy some number of units of demand $KCAP_k$, independent of product-type within a given time period.

Customers meanwhile are responsible for generation of end-item demand. The batch size that each customer orders Q_{cp} is to be fixed, that is a particular customer c orders Q_{cp} each time, although the batch size may vary from one customer to the next. The number of demand occurrences per unit time w_{cp}, however, varies according to some probability distribution $e(w_{cp})$. Customers

[1] A thorough discussion of the different types of pull systems, including Kanban and ConWIP, is presented in Hopp and Spearman (2000). The other pull systems presented were not included in the model since they follow material and information flows very similar to those of Kanban.

expect their orders to be filled no later than DUE_{cp} time periods from the time the order is placed and it is necessary that this deadline be met with minimum probability equal to some desired service level $SERV_{cp}$. The demand by the end customers is passed to the upstream echelons, generating subsequent demand at those levels.

The system under consideration is to be non-arborescent. Hopp and Spearman (2000) define arborescent supply chains to be such that each facility may receive shipments from no more than one other facility. The network topology can be seen as a whole in Figure 1.

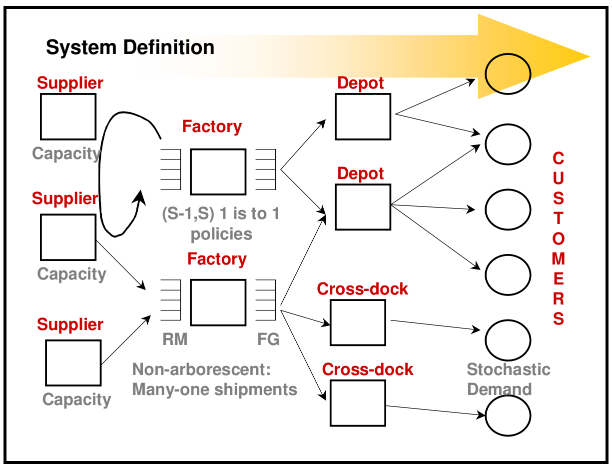

Figure 1: Network topology and location features

III.2 System Costs

The following costs, and their notations, are considered in the model.

CH^{RM}_{fr}	Annual holding cost per unit of part r at factory f
CH^{FG}_{fp}	Annual holding cost per unit of finished goods for product p at factory f
CH^{WIP}_{pf}	Annual holding cost per unit of work-in-process for product p at factory f
CH^{STOCK}_{dp}	Annual holding cost per unit of product p at depot d

CB_{cp}	Backorder cost per unit of product p per period for customer c
C^{proc}_{sr}	Per unit cost of purchasing part r from supplier s
C^{rep}_{rsf}	Fixed cost per replenishment of part r by supplier s for factory f
C^{ship}_{pfd}	Fixed cost per shipment of product p from factory f to depot d
C^{del}_{pdc}	Fixed cost per delivery of product p from depot d to customer c
$CAPX^{fact}_{f}$	Capital cost of opening factory f
$CAPX^{depot}_{d}$	Capital cost of opening depot d
OPX^{fact}_{f}	Operating cost of factory f per unit time
OPX^{depot}_{d}	Operating cost of depot d per unit time
C^{milk}_{f}	Fixed cost of setting up factory f for milk runs
C^{run}_{f}	Cost per unit collected by milk runs at factory f
C^{line}_{fp}	Cost of installing a line for product p in factory f

III.3 System Decisions

There are echelon-specific decisions to be made, specifically for the second and third echelons over which the supply chain manager has control, as well as those regarding the flow of parts and products through the entire supply chain. In the discussion to follow, variables that are relevant to each decision being presented are placed in parentheses. Those starting with *Y* are binary variables.

The major system decisions relating to the second echelon are the choice of factory sites to activate (Y^{fact}_{f}), and the mix of products that will be produced in each active factory ($Y^{fact\text{-}prod}_{fp}$). For each active factory, the number of lines of each product to install ($LINE_{pf}$) must be determined. For a particular factory, for a particular part-type, there is a choice to run either a Kanban make-to-stock line or a ConWIP make-to-ship line ($Y^{fact\text{-}pull}_{fp}$). Inventory decisions for factories include the capacity of the raw materials supermarket (RM_{fr}) and the finished goods supermarket (FG_{fp}). Another set of decisions involves the replenishment of the raw materials inventory. Here, in keeping with the principles of lean, the approach to be taken is not that of determining lot sizes and lead times, as in MRP. Instead, what is of importance is the period between which replenishments will be made ($PERIOD_{rsf}$). In addition, a choice is made between the modes of replenishment, that is, whether this will be done by direct shipments from the supplier or by milk runs (Y^{mode}_{rsf}). If a part is to be replenished via milk run, then additional relevant decisions involve the composition of the milk run and the sequence by which suppliers will be visited ($Y^{run}_{ss'f}$, Y^{beg}_{fs}, Y^{end}_{sf}).

Meanwhile, for the third echelon, the major decisions include the choice of depot sites and cross-dock sites to activate (Y^{depot}_{d}) and the mix of products to be carried by active depots or to be forwarded to active cross-docks ($Y^{depot\text{-}prod}_{dp}$). The latter forms the product set of the active depot or cross-dock, containing all the products for which a particular depot is to carry inventory and deliver to customers or simply the set of products for delivery straight to customer in the case of a cross-dock. The product sets of depots and cross-docks may overlap, meaning a particular product may be handled by more than one depot or cross-dock. Capacity of the depot in one-for-one replenishment would be equivalent to the base-stock level ($STOCK_{dp}$), to be assigned based on the product, and only to those product that are contained in the product set of the depot. Therefore, the capacity of the depot does not have to be equally distributed to all product types that it carries. For cross-docks meanwhile, although they are not supposed to carry inventory, each candidate cross-dock site is still assigned a variable $STOCK_{dp}$, just as the depots, but these are automatically set to zero by the cross-dock constraints discussed in the next section.

The activation of routes determines the flow of parts and products through the supply chain. A route is composed of one factory, one depot and one customer, and it is automatically activated if a product is shipped from the factory to the depot and delivered from the depot to the customer (Y^{route}_{pfdc}). In addition, a route may operate as either a Kanban route or a ConWIP route ($Y^{route\text{-}pull}_{pfdc}$).

IV. MODEL FORMULATION

Due to the numerous functions involved, the mathematical model is shown in its entirety in the Appendix, with each equation being labeled. Here, the overall structure of the model is discussed. Equation labels in parentheses following sections of the discussion refer to the functions of the model as labeled in the Appendix.

IV.1 Objective Function

The objective function (Equation A.1) is to be made up of the following costs:

- Holding costs - raw materials at factory
- Holding costs - finished goods at factory
- Holding costs – stock at depot
- Capital expenses– factories
- Capital expenses – depots
- Operating expenses – factories
- Operating expenses - depots
- Fixed milk run costs
- Variable milk run costs
- Backorder costs at the customers
- Procurement costs for raw materials
- Production line installation costs
- Direct replenishment fixed cost

- Shipment costs (transportation costs from factory to depot)
- Delivery costs (transportation costs from depot to customer)

IV.2 Constraints

The constraints may be classified into the following groups: service level, demand satisfaction, capacity, routes, inventory, transportation, milk runs, cross-docks and line installation

IV.2.1 Service Level

The service level of a route is related to the probabilities of two events, the delay and the default on delivery schedule (thus implying lateness), depending on the choice of pull system, Kanban or ConWIP. Meanwhile, these probabilities are functions of the variability in customer demand. This sub-section discusses the propagation of demand variability upstream then shows how these are included in the equations for delay and lateness.

The demand by the end customers is passed to the upstream echelons, generating consequent demand at those levels, which can be represented by the rates x^n_{jp}. This upstream flow of demand in a non-arborescent manner is modeled by the decision variable $PROP^n_{pjj'}$, representing the proportion of demand for product p at facility j' belonging to echelon n to be supplied by facility j belonging to echelon $n-1$. The binary variable $Y^{rel}_{pjj'}$ indicates whether facility j of echelon n replenishes the inventory of facility j' of echelon n+1. Using these decisions variables, the demand at facility j in echelon n can be expressed as:

$$x^n_{jp} = \sum_{j' \in ech\,(n+1)} PROP^n_{pjj'} x^{n+1}_{j'p} Y^{rel}_{pjj'} \qquad \forall j,\, p$$

(Equations A.8, A.10, A.14)

It follows then that the following constraint must also hold:

$$\sum_{j' \in ech\,(n+1)} PROP^n_{pjj'} = 1 \qquad \forall j,\, p$$

(Equations A.24-A.26)

As customer demand is a random variable, so too are the demand rates at each facility in the upstream echelons, each represented by some probability distribution function $f^n(x^n_{jp})$. Walpole (2000) discusses a method of transforming a probability distribution function of a random variable into a second distribution based on a second variable expressed as a function of the original random variable. This is used to derive the demand distributions of upstream echelons, where $f^n(x^n_{jp})$ is obtained through a transformation of $f^{n+1}(x^{n+1}_{jp})$. (Equations A.7, A.9, A.11, A.13)

A delay may come in three forms: a depot delay, a factory delay or a raw material delay. A depot delay occurs when at the time of an order, no stock is available at the depot assigned to serve the customer. Additional travel time is thus required to ship units from the finished goods supermarket of a factory to the depot. A factory delay occurs when even the finished goods supermarket of the factory is depleted, and so more time is required in waiting for the factory line to manufacture enough units to satisfy the order. The final delay is the raw material delay, which occurs when even the raw materials supermarket of the factory is depleted such that the system must wait for replenishment from a supplier. Regardless of the echelon to which a facility belongs, the delay function follows a similar structure all through out, as given below:

$$\int_0^{CRIT^n_{jp}} f(x^n_{jp})dx^n_{jp} = KDEL^n_{jp} \qquad \forall j, p$$

(Equations A.7, A.9, A.11, A.13)

In the above equation, the upper limit of integration, $CRIT^{n}_{jp}$, differs in form for disparate echelons and in value for different facilities within the echelon[2].

For a Kanban route, all three delay types matter. The scenario for ConWIP differs from kanban because for the former, orders must automatically incur the time for processing, the travel time from factory depot and from depot to customer. Thus, the only delay that matters in a ConWIP route is the raw material delay, and it only matters given that the basic cycle time for the ConWIP route, that is the process time plus travel time to the customer, is less than the time to due date.

Meanwhile, lateness is a function of the length of a delay $DELAY^{n}_{pj}$, given that the delay does occur. When a delay occurs, the length of the delay is dependent on the demand level at that facility. Thus, it will be necessary to transform the probability distribution function of demand at certain levels of the supply chain, $f(x^{n}_{jp})$, into the probability distribution function for the delay at those levels, again by the method described earlier in transforming $f(x^{n}_{jp})$ into $f(x^{n-1}_{j'p})$ for all n.

$$\int_0^{AVAIL^n_{pjj'j''...}} g(DELAY^n_{pjj'j''...})\,dDELAY^n_{pjj'j''...} = KSERV^n_{pjj'j''...} \quad \forall p, j, j', j'', ...$$

(Equations A.15, A.17, A.19, A.21)

Whereas delay probabilities are associated with facilities, service levels are associated with routes. The functional structure is also common for each echelon. Above, $g(DELAY^{n}_{pjj'j''...})$ is the probability distribution function of the length of a delay along the portion of a route starting with facilitiy j in echelon n, which replenishes facility j' in echelon $n+1$, which in turn replenishes facility j'' in echelon $n+2$, so on and so forth. The upper integration limit $AVAIL^{n}_{pjj'j''...}$ is the maximum amount of time available for the delay, which, like $CRIT^{n}_{jp}$, differs in form for disparate echelons and in value for different facilities within the echelon. Any delays that last longer than this limit constitute lateness. $KSERV^{n}_{pjj'j''...}$ meanwhile represents the resulting probability that an order for product p will still be on-time given that delays occur in each of the facilities j, j', j'' so on and so forth.

Figure 2 shows how the delay and lateness probabilities interact depending on the type of pull system employed. The branches that do not contain a single late occurrence at any level of the supply chain are on-time branches. Thus, the service level of a Kanban route without excess travel time is the sum of the probabilities of the following outcomes: there is no depot delay (that is, stock is readily available at the depot), a depot delay occurs but there is no factory delay (that is, stock is readily available at the finished goods supermarket of the factory), a depot delay and a factory delay occur but these are short and still not enough to render the system late while there is no raw material delay (that is, parts are readily available at the raw materials supermarket), or all previously mentioned types of delay occur but such is still not enough to render lateness while a supplier delay does not occur, or finally, all possible delays occur but such is not enough to render the system late. (Equation A.23)

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[2] The critical upper integration limits are derived from the Factory Physics chapter on Basic Factory Dynamics relationships

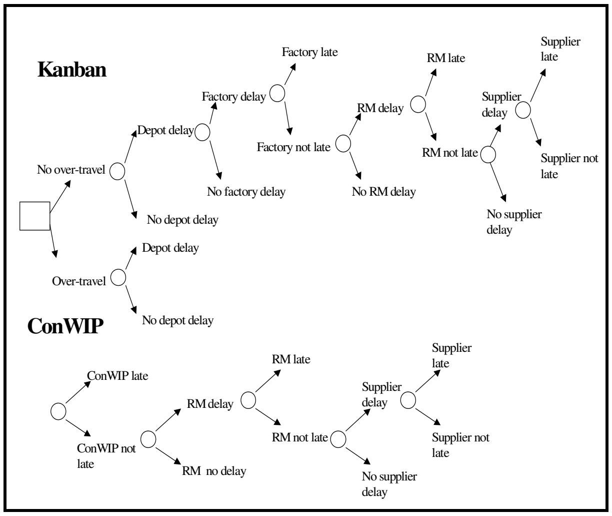

Figure 2: Kanban and ConWIP Delay and Lateness Probability Tree

IV.2.2 Demand Satisfaction

This ensures that demand at the customer level is propagated upstream in the supply chain. (Equations A.2, A.3)

IV.2.3 Capacity

There are two types of capacity constraints. First, the factory capacity constraint ensures that the number of standard production lines in a factory does not exceed the capacity of the factory site to hold lines. Second, the capacity constraint for the depot ensures that the base stock of a depot for all products assigned to the depot does not exceed the storage capacity of the depot. (Equations A.27, A.28)

IV.2.4 Routes

The route constraints serve to determine which routes have been chosen to operate by the model. A route for a product characterized by a factory, a depot and a customer, is automatically considered activated if that product is shipped from the factory to the depot and delivered from the depot to the customer (Equations A.29, A.31). Also, the route constraints classify all activated routes as operating either as a Kanban route or a ConWIP route (Equations A.32-A.34).

IV.2.5 Inventory

The inventory constraints ensure that inventory of finished goods are carried at facilities only if those facilities are activated and only if they are assigned to produce those finished goods. For raw materials, the inventory constraints for factories ensure that raw materials are held only if a factory is assigned to produce finished goods that consist of those raw materials (Equations A.35-A.46). In addition, the constraints also allow for carrying of finished goods by factories only if the factories operate under Kanban for those products (Equation A.47)

IV.2.6 Transportation

The transportation constraints ensure that products flow through a route only if the facilities comprising the route are assigned to carry the products concerned (Equations A.48-A.52).

IV.2.7 Milk Runs

The milk runs constraints govern the behavior of milk runs if they are chosen to operate. The structure of the milk run constraints is a modification of the classical traveling salesman model (Equations A.53-A.64).

IV.2.8 Cross-docks

These constraints ensure that cross-docks, meaning those facilities *d* for which $d \in cdock$, do not stock inventory of any product (Equation A.65). This is done since the formulation for cross-docks in very similar to that of the depot, except for the restriction that cross-docks should not carry inventory.

IV.2.9 Line Installations

This ensures that production lines are installed whenever products are assigned to factories (Equation A.67).

V. MODEL VALIDATION

Figure 3 presents the specific values of the model parameters used for model validation pictographically.

There are three candidate locations for factories and two candidate locations for third echelon facilities, one of which is a depot and the other is a cross-dock Both the capital and operating costs of the cross-dock are set to significantly lower values than those of the depot. The system is to service four customers, each with differing levels of demand for each product, different times to due date. They are however assigned similar service level requirements – at least 70% on time for Product 1 and at least 80% on time for Product 2.

Figure 4 is a diagram of the validation output. Only those facilities opened by the model are shown, as well as the final values of the different decision variables associated with each facility.

Only Factory 1 is opened. It receives raw material replenishment from all suppliers. These are made every 100, 42 and 25 periods for Part 1, Part 2 and Part 3 respectively. Every unit of Part 1 goes straight into the lines for both products, while units of Part 2 are stored in the raw materials supermarket, which is to have an inventory cap of 150 units. The same is true for Part 3. However, the cap is set much higher at 1000 units.

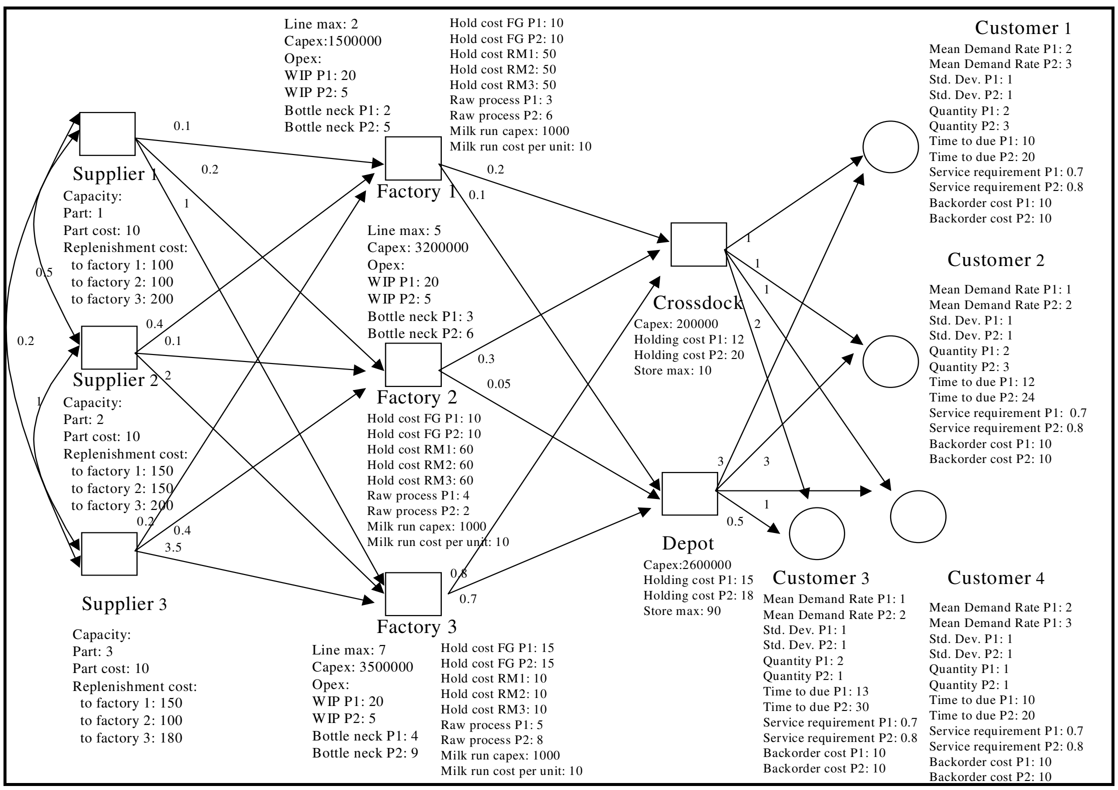

Figure 3: Diagram of Validation Input

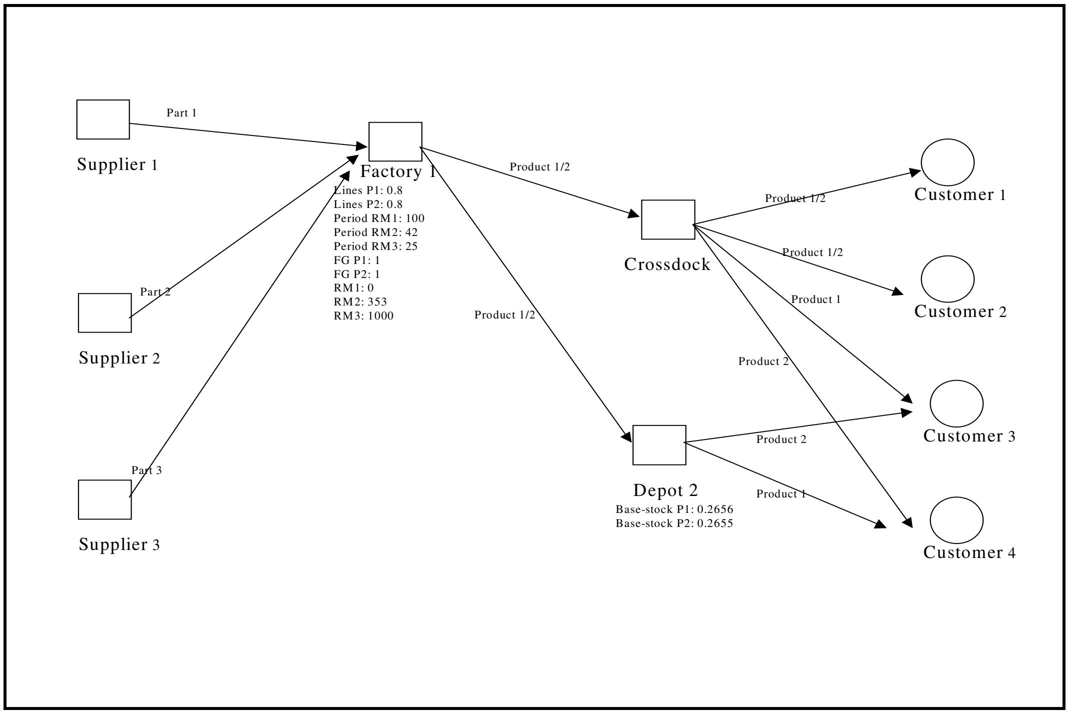

Figure 4: Diagram of Validation Output

Factory 1 supplies both the cross-dock and the depot. The base-stock level at the depot is 0.2656 for Product 1 and 0.2655 for Product 2. These are relatively small in value. Such makes the operation of the depot an approximation of the cross-dock operations. This is an indication that the supply chain manager should explore the option of considering the location for a cross-dock instead of a depot. The latter entails higher capital and operating costs than the former. The cross-dock delivers Product 1 to Customers 1, 2 and 3, while deliveries made to Customer 4 are composed of Product 2. Meanwhile, the Depot is to handle the demand of Customer 3 for Product 2 and that of Customer 4 for Product 1.

All routes are to operate under the ConWIP framework.

VI. CONCLUSIONS AND RECOMMENDATIONS

This study was borne of an unmet need for a supply chain model that integrates decisions on site selection, inventory allocation and shipment scheduling in a multi-echelon environment with multiple product-types. Furthermore, the model developed incorporates the use of mechanisms consistent with the principles of lean logistics.

There are several directions that can be recommended if one would like to extend this study. First, this study assumed that the third echelon is made up of depots or cross-docks. Further studies could explore the possibility of delivering to customers straight from the factory. This can be done without any modification in the model. Instead, the inputs to the model may be made such that dummy depots are established, one for each factory. The base-stock levels of these dummy depots may be forced to zero, as in a cross-dock, and the transportation from the factory to these depots may also be set at zero. Such a representation is the equivalent of allowing the possibility to direct delivery to customers from factories and is appealing since it does not require any major reformulation.

Second, this study considered the effect of demand-side variability on lean systems whereas it is also possible to consider supply-side variability, perhaps by modeling stochastic supplier lead times. A third option has to do with using the model to analyze lean supply chains in general. More numerical examples could be generated to show the particular instances that the optimal solution gravitates towards a lean supply chain as defined by advocates in the literature.

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Appendix: Complete Mathematical Model

Decision Variables

$STOCK_{dp}$	Base-stock level at depot d for product p	Units
FG_{fp}	Number of units of product p to be stocked in the finished goods supermarket at factory f, given that it operates under Kanban make-to-stock	Units
RM_{fr}	Number of units of part r to be stocked in the raw materials supermarket of factory f, regardless of pull system	Units
$PERIOD^{dir}{}_{rf}$	Frequency by which part r is replenished via direct shipment to factory f	Time
$PERIOD^{milk}{}_{rf}$	Frequency by which part is replenished r via milk run to factory f	Time
$LINE_{pf}$	Number of lines of product p to be installed into factory f	Lines
$PROP^{depot}{}_{pdc}$	Proportion of demand of customer c for product p to be handled by depot d	Dimensionless Proportion
$PROP^{fact}{}_{fdp}$	Proportion of demand of depot d for product p to be handled by factory f	Dimensionless Proportion
$PROP^{sup}{}_{rsf}$	Proportion of demand of factory f for part r to be handled by supplier s	Dimensionless Proportion

Binary Variables

$Y^{rep}{}_{rsf}$	1 if part r at factory f is replenished from supplier s and 0 otherwise
$Y^{ship}{}_{pfd}$	1 if product p is to be shipped from factory f to depot d to satisfy demand from customer c and 0 otherwise
$Y^{del}{}_{pdc}$	1 if product p is to be delivered from depot d to customer c and 0 otherwise
$Y^{fact\text{-}pull}{}_{fp}$	1 if product p is to be produced at factory f under Kanban and 0 if product p is to be produced at factory f under ConWIP, given that product p is to be produced at factory f
$Y^{fact}{}_{f}$	1 if factory f is to be opened and 0 otherwise
$Y^{fact\text{-}prod}{}_{fp}$	1 if factory f is to manufacture product p and 0 otherwise
$Y^{depot}{}_{d}$	1 if depot d is to be opened and 0 otherwise
$Y^{depot\text{-}prod}{}_{dp}$	1 if depot d is to carry product p and 0 otherwise
$Y^{milk}{}_{sf}$	1 if supplier s is in the milk run set of factory f and 0 otherwise
$Y^{mode}{}_{rsf}$	1 if replenishment of part r at factory f by supplier s is to be done via milk run, and 0 if replenishment of part r at factory f by supplier s is to be done via direct shipment
$Y^{milkq}{}_{f}$	1 if factory f conducts milk runs and 0 otherwise
$Y^{run}{}_{ss'f}$	1 if the milk run conducted by factory f directly links supplier s and supplier s' and 0 otherwise
$Y^{beg}{}_{fs}$	1 if the milk run conducted by factory f has supplier s as its first stop and 0 otherwise
$Y^{end}{}_{sf}$	1 if the milk run conducted by factory f has supplier s as its last stop and 0 otherwise
$Y^{route}{}_{pfdc}$	1 if product p passes through the route from factory f to depot d to customer c and 0 otherwise

$Y^{route\text{-}pull}_{pfdc}$	1 if product p passes through the route from factory f to depot d to customer c via the Kanban framework and 0 otherwise
$Y^{fact\text{-}part}_{rf}$	1 if factory f carries part r and 0 otherwise
Y^{trav}_{pfdc}	1 if the route from factory f to depot d to customer c for a product p exceeds the time to due date given by customer c for product p and 0 otherwise

System Variables

$KSERV^{fact}_{pfdc}$	Probability that an order for product p will still be one-time given that both a depot d delay and a factory f delay have occurred
$KSERV^{raw}_{pfdcr}$	Probability that an order for product p will still be on-time given that a depot d delay, a factory f delay, and a raw material r delay have all occurred
$KSERV^{sup}_{spfdcr}$	Probability that an order of part r for product p will still be on-time given that a depot d delay, a factory f delay, a raw material r delay, and a supplier s delay have all occurred
$KDEL^{depot}_{dp}$	Probability that no depot d delay for product p occurs
$KDEL^{fact}_{fp}$	Probability that no factory f delay for product p occurs given that a depot delay occurs
$KDEL^{raw}_{fr}$	Probability that no raw material delay for part r occurs given that a depot delay and factory f delay occur
$SERV^{con}_{pfdc}$	Probability that an order for product p will be on-time, given that it is assigned to a ConWIP route consisting of factory f, depot d and customer c
$WAIT_{fr}$	Wait time for the replenishment of raw material r at factory f in the instance of a backorder
REP_{rf}	Weighted average replenishment time overall suppliers providing factory f with part r
REP_{rsf}	Time between consolidation of orders for part r at supplier s and subsequent replenishment at factory f in case of direct shipment, or the time between departure and return of a milk run
$PERIOD_{rf}$	Number of time periods between replenishment of part r by factory f regardless of mode
$MILK_f$	Length of time consumed for a milk run by factory f
$FREQ^{milk}_{rf}$	Inverse of $PERIOD^{milk}_{rf}$, represents number of milk runs conducted for part r at factory f per period
$FREQ^{dir}_{rf}$	Inverse of $PERIOD^{dir}_{rf}$, represents number of direct replenishments made for part r at factory f per period
$FREQ_{rf}$	Inverse of $PERIOD_{rf}$, represents number of replenishments made for part r at factory f per period regardless of mode

Random Variables and their probability distribution functions

w_{cp}	Number of demand occurrences per unit time
$e(w_{cp})$	Probability distribution function for the number of demand occurrences per unit time
x^{depot}_{dp}	Demand rate for product p at depot d
$f(x^{depot}_{dp})$	Probability distribution function for the demand rate for product p at depot d

x^{fact}_{fp}	Demand rate for product p at factory f
$h(x^{fact}_{fp})$	Probability distribution function for the demand rate for product p at factory f
x^{raw}_{fr}	Demand rate for part r by factory f
$i(x^{raw}_{fr})$	Probability distribution function for the demand rate for part r by factory f
x^{sup}_{sr}	Demand rate for part r at supplier s
$j(x^{sup}_{sr})$	Probability distribution function for the demand rate for part r at supplier s

$$
\begin{aligned}
&Min\, Z = \sum_d \sum_p CH^{STOCK}_{dp} STOCK_{dp} + \sum_f \sum_r CH^{RM}_{fr} RM_{fr} + \sum_f \sum_p CH^{FG}_{fp} FG_{fp} + \\
&\sum_f CAPX^{fact}_f Y^{fact}_f + \sum_d CAPX^{depot}_d Y^{depot}_d + \sum_f OPX^{fact}_f CONV^{horizon} Y^{fact}_f \\
&+ \sum_d OPX^{depot}_d CONV^{horizon} Y^{depot}_d + \sum_f C^{milk}_f Y^{milk\,?}_f + \sum_r \sum_f C^{run}_f CONV^{horizon} E(X^{raw}_{fr}) Y^{\mathrm{mod}\,e}_{rf} \\
&\qquad CB_{cp} Q_{cp} E(X_{cp}) CONV^{year} [1 - (KDEL^{depot}_{dp} + (1 - KDEL^{depot}_{dp})(KDEL^{fact}_{fp}) + \\
&\qquad (1 - KDEL^{depot}_{dp})(1 - KDEL^{fact}_{fp}) KSERV^{fact}_{pfdc} KDEL^{raw}_{fr} + \\
&\sum_p \sum_f \sum_d \sum_c \sum_{r \in bom(p)} (1 - KDEL^{depot}_{dp})(1 - KDEL^{fact}_{fp}) KSERV^{fact}_{pfdc} (1 - KDEL^{raw}_{fr}) KSERV^{raw}_{fr} KDEL^{\sup}_{sr} \\
&\qquad + (1 - KDEL^{depot}_{dp})(1 - KDEL^{fact}_{fp}) KSERV^{fact}_{pfdc} (1 - KDEL^{raw}_{fr}) KSERV^{raw}_{fr} \\
&\qquad (1 - KDEL^{\sup}_{sr}) KSERV^{\sup}_{spfdcr})] Y^{routepull}_{pfdc} (1 - Y^{trav}_{pfdc}) \\
&\sum_p \sum_f \sum_d \sum_c CB_{cp} Q_{cp} E(X_{cp}) CONV^{horizon} (1 - KDEL^{depot}_{dp}) Y^{routepull}_{pfdc} Y^{trav}_{pfdc} \\
&\qquad CB_{cp} Q_{cp} E(X_{cp}) CONV^{horizon} \\
&\qquad [1 - (SERV^{con}_{pfdc} KDEL^{raw}_{fr} + SERV^{con}_{pfdc} (1 - KDEL^{raw}_{fr}) KSERV^{raw}_{pfdcr} \\
&\sum_p \sum_f \sum_d \sum_c \sum_{r \in bom(p)} KDEL^{\sup}_{sr} + SERV^{con}_{pfdc} (1 - KDEL^{raw}_{fr}) KSERV^{raw}_{pfdcr} \\
&\qquad (1 - KDEL^{\sup}_{sr}) KSERV^{\sup}_{spfdcr}](1 - Y^{routepull}_{pfdc}) \\
&+ \sum_r \sum_{s \in \sup(r)} \sum_f C^{proc}_{sr} CONV^{horizon} E(X^{raw}_{fr}) Y^{rep}_{rsf} + \sum_r \sum_{s \in bom(r)} \sum_f C^{rep}_{rsf} CONV^{horizon} FREQ^{dir}_{rf} \\
&+ \sum_f \sum_p C^{line}_{fp} LINE_{pf} + \sum_p \sum_f \sum_d C^{ship}_{pfd} E(X^{depot}_{dp}) PROP^{fact}_{fdp} CONV^{horizon} Y^{ship}_{pfd} \\
&+ \sum_p \sum_d \sum_c C^{del}_{pdc} E(X_{cp}) PROP^{depot}_{pdc} CONV^{horizon} Y^{del}_{pdc}
\end{aligned}
$$

Equation A.1

Demand Satisfaction Constraints

$$\sum_d Y^{del}_{pdc} \geq 1 \qquad \forall c, p$$ Equation A.2

$$\sum_f Y^{ship}_{pfd} \geq Y^{factprod}_{fp} \qquad \forall f, p$$ Equation A.3

Service Level Constraints

$$\sum_d TRAV_{dc} Y^{del}_{pdc} \leq DUE_{cp} \qquad \forall c, p$$ Equation A.4

$$(TRAV_{dc} + TRAV_{fd}) Y^{routepull}_{pfdc} \leq DUE_{cp} + MY^{TRAV}_{pfdc} \qquad \forall p, f, d, c$$ Equation A.5

$$RPT_{pf} + TRAV_{fd} + TRAV_{dc} \leq DUE_{cp} + MY^{route}_{pfdc} \qquad \forall p, f, d, c$$ Equation A.6

$$\int_0^{\frac{STOCK_{dp}-1}{\sum_f TRAV_{fd} Y_{pfd}^{ship}}} f(x_{dp}^{depot})dx_{dp}^{depot} = KDEL_{dp}^{depot} \qquad \forall d, p$$ Equation A.7

where $x_{dp}^{depot} = \sum_c Q_{cp} PROP_{pdc}^{depot} w_{cp} Y_{pdc}^{del} \qquad \forall d, p$ Equation A.8

$$\int_0^{\frac{FG_{fp}-1}{\left(\mu_{fp}^{fact} / BOTL_{pf}\right)}} h(x_{fp}^{fact})dx_{fp}^{fact} = KDEL_{fp}^{fact} \qquad \forall f, p$$ Equation A.9

where $x_{fp}^{fact} = \sum_d PROP_{fdp}^{fact} x_{dp}^{depot} Y_{pfd}^{ship} \qquad \forall f, p$ Equation A.10

$$\int_0^{RM_{fr} / \sum_s PERIOD_{rsf} Y_{rsf}^{rep}} i(x_{fr}^{raw})dx_{fr}^{raw} = KDEL_{fr}^{raw} \qquad \forall f, r$$ Equation A.11

where $x_{fr}^{raw} = \sum_{p \in r} x_{fp}^{fact} REQ_{rp} \qquad \forall f, r$ Equation A.12

$$\int_0^{CAP_{rs}^{\sup}} j(x_{rs}^{\sup})dx_{rs}^{\sup} = KDEL_{sr}^{\sup} \qquad \forall r, s$$ Equation A13

where $x_{rs}^{\sup} = \sum PROP_{rsf}^{\sup} x_{fr}^{raw} Y_{rsf}^{rep}$ Equation A.14

$$\int_0^{DUE_{cp}-TRAV_{fd}-TRAV_{dc}} k(DELAY_{pfd}^{kan-f})dDELAY_{pfd}^{kan-f} = KSERV_{pfdc}^{fact} \qquad \forall p, f, d, c$$
Equation A.15

where $k(DELAY_{pfd}^{can-f}) = h(DELAY_{pfd}^{kan-f} BOTL_{pf})BOTL_{pf}$ Equation A.16

$$\int_0^{DUE_{cp}-\frac{\mu_{cp}}{BOTL_{pf}}-TRAV_{fd}-TRAV_{dc}} l(DELAY_{rf}^{raw})dDELAY_{rf}^{raw} = KSERV_{pfdcr}^{raw} \qquad \forall r, f, p \in r, d, c$$
Equation A.17

where
$$l(DELAY_{rf}^{kan-raw}) = i\left(\frac{RM_{fr}}{DELAY_{rf}^{kan-raw} - \sum_s FREQ_{rsf} Y_{rsf}^{rep} - REP_{rf}}\right)\left[\frac{-RM_{fr}}{\left(DELAY_{rf}^{kan-raw} - \sum_s FREQ_{rsf} Y_{rsf}^{rep} - REP_{rf}\right)^2}\right]$$
Equation A.18

$$\int_0^{DUE_{cp}-TRAV_{dc}-TRAV_{fd}} m(LT_{pf}^{con})dLT_{pf}^{con} = SERV_{pfdc}^{con} \qquad \forall p, f, d, c$$ Equation A.19

where $m(LT_{pf}^{con}) = h\left(\frac{LT_{pf}^{con} RPT_{pf}}{BOTL_{pf}}\right)\frac{RPT_{pf}}{BOTL_{pf}}$ Equation A.20

$$\int_0^{DUE_{cp}-\mu_{cp}/BOTL_{pf}-TRAV_{fd}-TRAV_{dc}-TRAV_{sf}} n(DELAY_{sr}^{\sup})dDELAY_{sr}^{\sup} = KSERV_{spfdcr}^{\sup}$$ Equation A.21
$\forall s, p, f, d, c, r$

where $n(DELAY_{sr}^{\sup}) = j(DELAY_{sr}^{\sup} CAP_{rs}^{\sup}) CAP_{rs}^{\sup}$ Equation A.22

$$KDEL_{dp}^{depot} + (1 - KDEL_{dp}^{depot})(KDEL_{fp}^{fact}) +$$
$$(1 - KDEL_{dp}^{depot})(1 - KDEL_{fp}^{fact}) KSERV_{pfdc}^{fact} KDEL_{fr}^{raw} +$$
$$(1 - KDEL_{dp}^{depot})(1 - KDEL_{fp}^{fact}) KSERV_{pfdc}^{fact} (1 - KDEL_{fr}^{raw}) KSERV_{fr}^{raw} KDEL_{sr}^{\sup} +$$
$$(1 - KDEL_{dp}^{depot})(1 - KDEL_{fp}^{fact}) KSERV_{pfdc}^{fact} (1 - KDEL_{fr}^{raw}) KSERV_{fr}^{raw} (1 - KDEL_{sr}^{\sup}) KSERV_{spfdcr}^{\sup}$$
$$\geq SERV_{cp} - M(2 - Y_{pfdc}^{routepull} - Y_{pfdc}^{trav}) \qquad \forall p, f, d, c, r \in bom(p)$$
$$KDEL_{dp}^{depot} \geq SERV_{cp} - M\left(2 - \sum_{f} (Y_{pfdc}^{routepull} + (1 - Y_{pfdc}^{trav}))\right) \qquad \forall p, f, d, c$$ Equation A.23

Non-arborescence Constraints

$\sum_{d} PROP_{pdc}^{depot} = 1 \qquad \forall c, p$ Equation A.24

$\sum_{f} PROP_{fdp}^{fact} = 1 \qquad \forall d, p$ Equation A.25

$\sum_{s \in \sup(r)} PROP_{rsf}^{\sup} = 1 \qquad \forall r, f$ Equation A.26

Capacity Constraints

$\sum_{p} CONV_{p}^{store} STOCK_{dp} \leq STORE_{d}^{\max} \qquad \forall d$ Equation A.27

$\sum_{p} CONV_{p}^{line} LINE_{pf} \leq LINE_{f}^{\max} \qquad \forall f$ Equation A.28

Route Constraints

$1 + Y_{pfdc}^{route} \geq Y_{pdc}^{del} + Y_{pfd}^{ship} \qquad \forall p, f, d, c$ Equation A.29

$Y_{pfdc}^{route} \leq Y_{pdc}^{del} \qquad \forall p, f, d, c$ Equation A.30

$Y_{pfdc}^{route} \leq Y_{pfd}^{ship} \qquad \forall p, f, d, c$ Equation A.31

$1 + Y_{pfdc}^{routepull} \geq Y_{pfdc}^{route} + Y_{fp}^{factpull} \qquad \forall p, f, d, c$ Equation A.32

$Y_{pfdc}^{routepull} \leq Y_{pfdc}^{route} \qquad \forall p, f, d, c$ Equation A.33

$Y_{pfdc}^{routepull} \leq Y_{fp}^{factpull} \qquad \forall p, f, d, c$ Equation A.34

Inventory Constraints

$STOCK_{dp} \leq MY_{dp}^{depotprod} \qquad \forall d, p$ Equation A.35

$RM_{fr} \leq MY_{rf}^{factpart} \qquad \forall f, r$ Equation A.36

$FG_{fp} \leq MY_{fp}^{factprod} \qquad \forall f, p$ Equation A.37

$$LINE_{pf} \leq MY_{fp}^{factprod} \qquad \forall f, p \qquad \text{Equation A.38}$$

$$\sum_{p} Y_{dp}^{depotprod} \leq MY_{d}^{depot} \qquad \forall d \qquad \text{Equation A.39}$$

$$\sum_{p} Y_{fp}^{factprod} \leq MY_{f}^{fact} \qquad \forall f \qquad \text{Equation A.40}$$

$$Y_{fp}^{factprod} \leq Y_{fr}^{factpart} \qquad f, p, r \in bom(p) \qquad \text{Equation A.41}$$

$$Y_{rsf}^{rep} \leq Y_{rf}^{factpart} \qquad \forall f, r, s \in \sup(r) \qquad \text{Equation A.42}$$

$$\sum_{s \in \sup(r)} Y_{rsf}^{rep} \geq Y_{fr}^{factpart} \qquad r, f \qquad \text{Equation A.43}$$

$$\sum_{p \in der(r)} Y_{fp}^{factprod} \geq Y_{rf}^{factpart} \qquad \forall r, f \qquad \text{Equation A.44}$$

$$\sum_{p} Y_{fp}^{factprod} \geq Y_{f}^{fact} \qquad \forall f \qquad \text{Equation A.45}$$

$$\sum_{p} Y_{dp}^{depotprod} \geq Y_{d}^{depot} \qquad \forall d \qquad \text{Equation A.46}$$

$$FG_{fp} \leq MY_{fp}^{factpull} \qquad \forall f, p \qquad \text{Equation A.47}$$

Transportation Constraints

$$\sum_{d} Y_{pfd}^{ship} \leq MY_{fp}^{factprod} \qquad \forall f, p \qquad \text{Equation A.48}$$

$$\sum_{f} Y_{pfd}^{ship} \leq MY_{dp}^{depotprod} \qquad \forall d, p \qquad \text{Equation A.49}$$

$$\sum_{f} Y_{pfd}^{ship} \geq Y_{dp}^{depotprod} \qquad d, p \qquad \text{Equation A.50}$$

$$\sum_{c} Y_{pdc}^{del} \geq Y_{dp}^{depotprod} \qquad \forall d, p \qquad \text{Equation A.51}$$

$$\sum_{d} Y_{pfd}^{ship} \geq Y_{fp}^{factprod} \qquad \forall f, p \qquad \text{Equation A.52}$$

Milk Run Constraints

$$REP_{rf} = \sum_{s} PROP_{rsf}^{\sup} REP_{rsf} \qquad \forall r, f \qquad \text{Equation A.53}$$

$$REP_{srf} = TRAV_{sf} Y_{rsf}^{\mathrm{mod}\,e} + MILK_{f} \qquad \forall r, s, f \qquad \text{Equation A.54}$$

$$\sum_{s \in \sup(r)} Y_{sf}^{milk} \leq MY_{rsf}^{\mathrm{mod}\,e} \qquad \forall r, f \qquad \text{Equation A.55}$$

$$\sum_{s' \neq s} (Y_{ss'f}^{run} + Y_{sf}^{end}) = Y_{sf}^{milk} \qquad \forall s, f \qquad \text{Equation A.56}$$

$$\sum_{s' \neq s} (Y_{s'sf}^{run} + Y_{fs}^{beg}) = Y_{sf}^{milk} \qquad \forall s, f \qquad \text{Equation A.57}$$

$$Y_{ss'f}^{run} \leq Y_{sf}^{milk} \qquad \forall s, s' \neq s, f \qquad \text{Equation A.58}$$

$$Y_{s'sf}^{run} \leq Y_{sf}^{milk} \qquad \forall s, s' \neq s, f \qquad \text{Equation A.59}$$

$$MILK_{f} = \sum_{s} \sum_{s' \notin s} Y_{ss'f}^{run} TRAV_{ss'} + \sum_{s} Y_{fs}^{beg} TRAV_{fs} + \sum_{s} Y_{sf}^{end} TRAV_{sf} \qquad \forall f \qquad \text{Equation A.60}$$

$$MILK_f \leq CONV^{day} \qquad \forall f$$ Equation A.61

$$\sum_{s \in \sup(r)} Y_{fs} = Y_f^{milkq} \qquad \forall f$$ Equation A.62

$$\sum_{s \in \sup(r)} Y_{sf} = Y_f^{milkq} \qquad \forall f$$ Equation A.63

$$\sum_r \sum_{s \in \sup(r)} Y_{rsf}^{\mathrm{mod}\,e} \leq MY_f^{milkq} \qquad \forall f$$ Equation A.64

Cross-dock constraints

$$STOCK_{dp} = 0 \qquad \forall d \in cdock(d), p$$ Equation A.65

Line Installation Constraints

$$Y_{fp}^{factprod} \leq LINE_{pf} \qquad \forall f, p$$ Equation A.66